

## Some Contraction Mappings: A Few Fixed Point Results On F-Metric Spaces

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### ABSTRACT

We have given a description of the fixed-point theorems in F-metric spaces for convex contraction mappings. Some definitions are introduced with the help of examples to make it more interested. We also introduced some examples to make our theoretic outcomes more clear.

**Keywords:** F-complete, F-convex contraction, F-metric space, sequentially compact, compact metric space.

### INTRODUCTION

In the section of functional analysis and real analysis the fixed-point theory is an essential apparatus. Theoretical framework of FPT in the metric spaces is a vast and valuable research field. The Banach introduced a principle known as the contraction mapping principle has also a great importance in field of research as it contains numerous applications. The Banach contraction mapping principle was later generalized and was called convex contraction principle.

Let  $Z$  be the set of function  $z(0, \infty) \rightarrow \mathbb{R}$ , meeting the criteria.  $(Z_1)$  is non-decreasing i.e.  $0 < s < t \Rightarrow z(s) \leq z(t)$  and  $(Z_2) \forall$  sequence  $\{t_n\} \subset (0, \infty)$ , we have  $\lim_{n \rightarrow +\infty} t_n$  if and only if  $\lim_{n \rightarrow +\infty} z(t_n) = -\infty$

**1.1 Definition:** we consider that  $Y$  be non-empty set. A  $D: Y \times Y \rightarrow [0, \infty)$  is function is known as a F- metric on  $Y \ni (z, \infty) \in Z \times [0, \infty)$  in such a way that for all  $s, t \in Y$  the below conditions are true

$$Z1. Z(x, y) = 0 \Leftrightarrow x = y$$

$$Z2. Z(x, y) = Z(y, x)$$

$$Z3. \forall n \in \mathbb{N}, n \geq 2 \text{ and } \forall \{u_i\}_i^n \subset X \text{ with } (u_1, u_n) = (x, y), \text{ we have}$$

$$Z(x, y) > 0 \Rightarrow f_{+\infty}(D(x, y)) \leq z \left( \sum_{i=1}^{n-1} D(u_i, u_{i+1}) \right)$$

So it is claimed that  $D$  is F- metric on  $Y$ , and the point  $(Y, 0)$  is known as F-metric space.

**1.Example:** Let  $Y = \mathbb{R}$  &  $M: Y \times Y \rightarrow [0, \infty)$  be defined as follows :

$$Z(x, y) = \begin{cases} e^{1x-1y}, & x \neq y \\ 0, & x = y \end{cases}$$

Thus  $M$  is said to be F-metric on  $Y$ . Since  $Z(1, 3) = e^2 \geq Z(1, 2) + Z(2, 3) = 2e$ , then  $Z$  is not metric on  $Y$ .

**2.Example:** Let  $Z = \mathbb{R}$  and  $D: Z \times Z \rightarrow [0, \infty)$  be specified as follows:

$$D(p, q) = \begin{cases} (p - q)^2, & p, q \in [0, 3] \times [0, 3] \\ |p - q|, & \text{otherwise} \end{cases}$$

**1.2 Definition :** Given a metric space, let's say  $(s, d)$ . The continuous self-maps "T" on  $X$  are referred to as a convex second order contraction of iff there exist  $a_i \in (0, 1), i = 1, 2$ , with  $a_1 + a_2 < 1$  such that  $\forall s, t \in X$

$$D(T^2s, T^2t) \leq a_1 d(Ts, Tt) + a_2 d(ds, t)$$

**Theorem 1.1** If  $(Y, d)$  is a whole metric space. As a result, every convex contraction mapping of order 2 has a fixed point that is distinct.

**1.3 Definition :** We consider  $(Y, d)$  to be a metric space. It is known as a two sided convex contraction mapping for the continuous self-map "T" on  $Y$ . if  $\exists a_i, b_i \in (0, 1), i = 1, 2$  with  $a_1 + b_1 + b_2 < 1$

1 such that for all  $s, t \in Y$ ,

$$D(T^2s + T^2t) \leq a_1d(s, T_s) + a_2d(T_s, T_t) + b_1d(t, T_t) + b_2d(T_t, T^2t).$$

Theorem 1.2 : We take the metric space  $(Y, d)$  to be entirely complete. Therefore, each fixed point in a contraction mapping with a two-sided convex contraction must be distinct.

**1.4Definition :** We presumptively use a metric space for  $(Y, d)$ . The term orbitally continuous at a point refers to a self-mapping  $T$  on  $Y$ .  $s^* \in Y$  if for any  $Y_n \subseteq O(s, T)$  we have

$$Y_n \rightarrow s^* \Rightarrow T_{Y_n} \rightarrow Ts^* \text{ as } n \rightarrow \infty$$

## 2. (CONVEX CONTRACTIONS) On F Metric spaces, a few mappings.

We have demonstrated a few fixed-point theorems for convex contraction mapping based on F-metric Spaces to support this.

**Theorem 2.1 :** We take the pair  $(y, 1)$  to be a whole F-metric space. Assume that  $T$  is an order 2-convex contraction mapping on  $Y$ . As a result,  $T$  must have a singular fixed point.

Proof : Assume  $Y_0$  be a point in  $Y$  which is arbitrary . We can determine a sequence  $\{Y_n\}$  as  $s_{n+1} = Ts_n$  for each  $n \in N \cup \{0\}$ .

If  $s_m = s_{m+1}$  for some  $n \in N \cup \{0\}$ , then it is clear that  $s_m$  is a fixed-point of  $T$ .

Then , we let  $s_n \neq s_{n+1} \forall n \in N \cup \{0\}$

Set  $v = \max \{D(s_0, Ts_0), D(Ts_0, T^2s_0)\}$  we have

$$D(T^3s_0, T^2s_0) \leq a_1D(T^2s_0, Ts_0) + a_2D(Ts_0, s_0) \leq v(a_1 + a_2)$$

Similarly ,

$$D(T^4s_0, T^3s_0) \leq a_1D(T^3s_0, T^2s_0) + a_2D(T^2s_0, Ts_0) \leq a_1v(a_1 + a_2) + a_2v$$

$$\leq v(a_1 + a_2),$$

$$D(T^5s_0, T^4s_0) \leq a_1D(T^4s_0, T^3s_0) + a_2D(T^3s_0, T^2s_0) \leq a_1v(a_1 + a_2) + a_2v(a_1 + a_2) \leq v(a_1 + a_2)^2$$

An induction argument shows that

$$D(T^{2m+1}s_0, T^{2m}s_0) \leq v(a_1 + a_2)^m \tag{2.1}$$

$$\text{And } D(T^{2m-1}s_0, T^{2m}s_0) \leq v(a_1 + a_2)^{m-1}, \forall m \in N. \tag{2.2}$$

Now we show that  $\{T^2s_0\}$  is a F-Cauchy sequence.

Let  $(f, \alpha) \in F \times [0, \infty)$  such that  $D_3$  is satisfied. Let  $\varepsilon > 0$  be fixed. From  $(F_2) \exists \delta > 0$  such that

$$0 < t < \delta \Rightarrow f(t) < f(\varepsilon) - \alpha \tag{2.3}$$

Let  $m, n \in N$  and  $n > m$ . If  $m = 2k$  or  $m = 2k+1$  from (2.1) & (2.2) we have  $\sum_{i=m}^{n-1} D(T^i x_0, T^{i+1}) \leq$

$$2v(a_1 + a_2)^k \left( \frac{1}{1 - (a_1 + a_2)} \right) = 0$$

Since  $a_1 + a_2 < 1$  we have

$$\lim_{k \rightarrow +\infty} 2v(a_1 + a_2)^k \left( \frac{1}{1 - (a_1 + a_2)} \right) = 0$$

Then, there exist some  $N \in N$  Such that

$$0 < 2v(a_1 + a_2)^k \left( \frac{1}{1 - (a_1 + a_2)} \right) < \delta$$

For all  $k \geq N$ . Using (2.3) and  $(F_1)$ , we get

$$F\left(\sum_{i=m}^n D(T^i s_0, T^{i+1} s_0)\right) \leq f 2v(a_1 + a_2)^k \left( \frac{1}{1 - (a_1 + a_2)} \right) \leq f(\varepsilon) - \alpha$$

From  $(D_3)$  & (2.4) for  $n > m \geq N$  we have

$$f(D(T^m s_0, T^n s_0)) \leq \sum_{i=m}^{n-1} D(T^i s_0, T^{i+1} s_0) + \alpha < f(\epsilon)$$

Using  $(F_1)$ , we obtain  $D(T^m s_0, T^n s_0) < \epsilon, n > m \geq N$ .

So  $\{s_n\}$  is F-cauchy in the F complete F metric space  $X$ . So  $\exists x^* \in X$  that is  $\lim_{n \rightarrow \infty} D(s_n, s^*) = 0$

Since T is continuous in F then we have ,

$$Ts^n = T(\lim_{n \rightarrow \infty} s_n) = \lim_{n \rightarrow \infty} Ts_n = s^*$$

The fixed point of T is the same. Finally, we will demonstrate that fixed point is singular.

Consider that another fixed point does exist, and  $D(s^*, t^*) > 0$

From 1.1 we have

$$D(s^*, t^*) = D(T^2 s^*, T^2 t^*) < a_1 D(T s^* T t^*) + a_2 D(s^*, t^*) \\ = (a_1 + a_2) D(s^*, t^*)$$

Since  $(a_1 + a_2) < 1$

We get ,  $s^* = t^*$

**Example 2.1 :** Let  $S = [0, \infty]$  be endowed with the F-metric

$$D(p, q) = \begin{cases} (p - q)^2, & p, q \in [0, 3] \times [0, 3] \\ |p - q|, & \text{otherwise} \end{cases}$$

Define  $T: p \rightarrow p$  by

$$T(p) = \frac{p}{2} + 1$$

Hence for  $a_1 = 0$  and  $a_2 = \frac{1}{2}$  Since every requirement has been met, T has a distinct fixed point in S.

**Theorem 2.2 :** Assume that  $(X, D)$  is an F-complete and an F-metric, and that T is a two-sided convex mapping on X. In that case, it has a special fixed point..

**Proof :** Assume any point  $s_0$  in the coordinate space X. The Picard iteration cycle is described  $\{s_n\}$  by  $s_{n+1} = Ts_n \forall n \in N \cup \{0\}$

We assume that  $s_n \neq s_{n+1} \forall n \in N \cup \{0\}$ . Set  $v = \max \{D(s_0, s_1), D(Ts_0, T^2 t_0)\}$

From 1.2, we have

$$D(T^3 s_0, T^2 s_0) \leq a_1 D(Ts_0, T^2 s_0) + a_2 D(T^2 s_0, T^3 s_0) + b_1 D(s_0, Ts_0) + b_2 D(Ts_0, T^2 s_0)$$

Then we have ,

$$D(T^3 s_0, T^2 s_0) \leq \frac{\lambda}{\gamma} v$$

Where  $\lambda = a_1 + b_1 + b_2$  and  $\gamma = 1 - a_2$

Similarly ,  $D(T^4 s_0, T^3 s_0) \leq \frac{\lambda}{\gamma} v$ ,

And  $D(T^5 s_0, T^4 s_0) \leq \left(\frac{\lambda}{\gamma}\right)^2 v$

Continuing these results we get ,

$$D(T^{2m+1} s_0, T^{2m} s_0) \leq \left(\frac{\lambda}{\gamma}\right)^m v \quad (2.5)$$

$$D(T^{2m-1} s_0, T^{2m} s_0) \leq \left(\frac{\lambda}{\gamma}\right)^{m-1} v \quad (2.6)$$

Now, we array that  $\{T^n s_0\}$  is a F Cauchy sequence .

Let  $m, n \in N$  and  $n > m$ . If  $m = 2k$  or  $m = 2k+1$  from (2.5) & (2.6) we have

$$\sum_{i=m}^{n-1} D(T^i s_0, T^{i+1} s_0) \leq 2v \left(\frac{\lambda}{\gamma}\right)^k \left(\frac{1}{1-\frac{\lambda}{\gamma}}\right)$$

Since  $\frac{\lambda}{\gamma} < 1$ , we have  $\lim_{k \rightarrow +\infty} 2v \left(\frac{\lambda}{\gamma}\right)^k \left(\frac{1}{1-\frac{\lambda}{\gamma}}\right) = 0$

$\therefore \{X_n\}$  is a F Cauchy sequence in F-complete F-metric.

Since T is continuous in F from theorem 2.1

$$Ts^*T = T\left(\lim_{n \rightarrow \infty} x_n\right) = \lim_{n \rightarrow \infty} Ts_n = s^*$$

So,  $s^*$  is the fixed point T. To explain the uniqueness of the fixed point  $s^*$ , let  $z^*$  is one more fixed-point of T and  $D(s^*, z^*) > 0$  from (1.2) we show,

$$D(x^*, z^*) = D(T^2s^*, T^2z^*) \leq a_1D(s^*, Ts^*) + a_2D(Ts^*, T^2s^*) + b_1D(z^*, Tz^*) + b_2D(Tz^*, T^2z^*) \\ \leq (a_1 + a_2 + b_1 + b_2) D(s^*, z^*)$$

Since,  $a_1 + a_2 + b_1 + b_2 < 1$

$$D(s^*, z^*) = 0 \implies s^* = z^*$$

Definition 2.1 : Let's assume  $(Z, D)$  be a F-metric space and  $T : s \rightarrow Z$  be cyclic  $(\alpha, \beta)$ -admissible mapping we say that T is a  $(\alpha, \beta)$ - If T is an orbitally continuous and there is a contraction, it is two-sided. and their exist  $a, b_i \in (0, 1), i=1, 2$ , such that

$$\alpha(x)\beta(y) \geq 1 \implies D(T^2s, T^2t) \leq a_1D(s, Ts) + a_2D(Ts, T^2t) + b_1D(t, Tt) + b_2D(Tt, T^2t)$$

Where  $a_1 + a_2 + b_1 + b_2 < 1 \forall x, y \in X$

Example 2.2.

Let,

$$T_X = \begin{cases} \frac{-x}{3}, & s \in [-3, 3] \\ s^3, & \text{otherwise} \end{cases}$$

And  $\alpha, \beta : X \rightarrow [0, +\infty)$  be given by

$$\alpha(x) = \begin{cases} 1, & s \in [-3, 3] \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \beta(x) = \begin{cases} 1, & s \in [-3, 3] \\ 0, & \text{otherwise} \end{cases}$$

Let  $x \in X$ , if  $\alpha(s) \geq 1$ , then  $s \in [-3, 0]$

And so  $T(s) \in [0, 3]$ , that is  $\beta(T_x) \geq 1$ . Also if  $\beta(x) \geq 1$ , Then  $\alpha(T_x) \geq 1$ . Thus T is a cyclic  $(\alpha, \beta)$  admissible mapping. Let  $s, t \in s$  &  $\alpha(s)$  and  $\beta(t) \geq 1$ . Then  $s \in [-3, 0]$  and  $t \in [0, 3]$ . We get  $D(T^2s, T^2t) = D\left(\frac{s}{9}, \frac{t}{9}\right) = 1$

$$\leq \frac{1}{2} D(s, t)$$

$\therefore$  For  $a_1=0$  &  $a_2 = \frac{1}{2}$  this satisfies all conditions. Therefore there is a unique fixed point of T.

### Results on topology of F-metric space

We have now expanded F-metric spaces topologically. Lindelof F-metric spaces are at least what we assume these metric spaces to be.

**Theorem 3.1** :-

Every Lindelof F is separable.

**Proof** :- Assume  $(Z, d)$  be Lindelof F metric spaces, and  $A_n = \{ \text{int} ( B(s, \frac{1}{n}) : s \in Z ) \}$ . Then  $A_n$  is an open cover of Z for each  $n \in N$ . Since,  $(Z, d)$  is Lindelof,  $A_n$  has a countable subcover say  $A_n = \{ \text{int} ( B(s_{ni}, \frac{1}{n}) ) : i \in N \}$  for all  $n \in N$

Let,  $D = \{ s_n : i, n \in N \}$  then D belongs to Z as a countable subset.. Next, show that  $\bigcup_D = Z$ . Let,  $s \in Z$  be arbitrary and in Z, u is an open set conveying Z. . Then  $\exists r > 0$  to some extent that  $s \in B(s, r) \subset u$ . Let, us choose  $n \in N$  such that  $\frac{1}{m} < r$ . Since  $A'_m = \{ \text{int} ( B(s_m, \frac{1}{m}) ) : i \in N \}$  must be an open cover of s,  $s \in \{ \text{int} ( B(s_m, \frac{1}{m}) ) \}$  for some  $k \in N$

$$\therefore d(s_{mk}, s) < \frac{1}{m} < r \implies s_{mk} \in (B(s, r)) \subset u.$$

Let us choose  $n \in N$  such that  $\frac{1}{m} < r$ . Since  $A'_m = \{ \text{int} ( B(s_m, \frac{1}{m}) ) : i \in N \}$  is an open cover of s,  $Z \in$

$\{ \text{int} ( B(s_m, \frac{1}{m}) ) \}$  for some  $k \in N$

$\therefore d (s_{mk}, s) < \frac{1}{m} < r \implies x_{mk} \in (B(x, r)) \subset u$ .

Also,  $s_{mk} \in D$ . Thus  $D \cap U \neq \emptyset$ , hence  $s \in \bar{D}$ . So  $\bar{D} = Z$  and consequently  $Z$  is separable.

**Theorem 3.2 :** The second countability of every Lindelof F-metric space.

**Proof :** Assume  $(Z, d)$  be Lindelof F-metric space and  $A_n = \{ \text{int} ( B(s, \frac{1}{n}) : s \in Z \} \forall n \in N$ .

Then  $A_n$  is an open cover of  $Z$  for each  $n \in N$ . Since,  $(Z, d)$  is Lindelof,  $A_n$  has a countable subcover say  $A_n = \{ \text{int} ( B(s_{ni}, \frac{1}{n}) : i \in N \} \forall n \in N$

Let,  $A = \{ \text{int} ( B(s_{ni}, \frac{1}{n}) : i, n \in N \}$  then open sets in  $Z$  are gathered in  $A$ , a countable collection. In order to demonstrate that  $A$  is a base for  $Z$ 's topology, and  $x \in u$ . So there exist  $r > 0 : x \in B(s, r) \subset u$ .

Let us choose  $n \in N$  such that  $\frac{1}{n} < r$ . Then,  $B(s, \frac{1}{n}) \subset B(s, r) \subset u$ .

Then by  $(F_2)$  there exists  $\delta > 0 :$

$0 < t < \delta \implies$

$f(t) < f(\frac{1}{n}) - a \rightarrow (*)$

Again, choose  $m \in N$  such that  $\frac{1}{m} < \delta$ . Since  $A'_{2m}$  is an open cover of  $s$ ,  $s \in \text{int} ( B(s_{2m}, \frac{1}{2m}) )$  for some  $i \in N$ .

Let,  $s \in B(s_{2m}, \frac{1}{2m})$

$$d(t, s_{2m}) + d(s_{2m}, s) < \frac{1}{2m} + \frac{1}{2m} = \frac{1}{m} < \delta$$

So by \*

$$f(d(t, s_{2m}) + d(s_{2m}, s)) < f(\frac{1}{n}) - a$$

If  $t=s$  then  $t \in B(s, \frac{1}{n}) \subset u$

If not then by (DB)

$$f(d(t, s_{2m}) + d(s_{2m}, s)) + a \geq f(d(s, t))$$

$$f(d(s, t)) \leq f(\frac{1}{n}) - a + a \leq f(\frac{1}{n})$$

$$\implies d(s, t) \leq \frac{1}{n}$$

$$\implies t \in B(s, \frac{1}{n}) \subset u$$

$$\therefore B(s_{2m}, \frac{1}{2m}) \subset u.$$

$$\implies s \in \text{int} ( B(s_{2m}, \frac{1}{2m}) )$$

$$\implies s \in \text{int} ( B(s_{2mi}, \frac{1}{2m}) ) \subset u.$$

Where  $\text{int} ( B(s_{2mi}, \frac{1}{2m}) ) \in A$ .

$\therefore$  The topology on  $Z$  has a countable base, hence  $(Z, d)$  is second countable.

**Definition 3. 1 :** Sequentially compact :-  $Y$  is sequentially compact if every sequence of its points has a convergent subsequence that converges to one of its points.

**Definition 3. 2:-** If there is a finite sub-cover for every open cover in  $Y$  then  $Y$  is compact metric space.

**Theorem 3. 3 .** Each and every subset of sequentially compact F-metric space, which is also Lindelof, is

compact.

**Proof** :- From theorem 3.1 we have proved that every Lindelof F – metric space is separable , this indicates that the pair (s,d) comprises a dense countable subset of A.

Assume the collection of open balls with rational radius containing center in A be known as B . Since A is countable and the rationals are also countable , B must be countable . Assume that the sub-collection of balls in B that are present in at least one of the open sets in the convex Sa is referred to as C.. Since C be the subset of B ,  $\therefore C$  is countable.

For every  $s \in X$  , there is a Sa such that  $s \in Sa$  . Since Sa is open there exists an  $\varepsilon > 0$  such that  $B_\varepsilon(s) \subseteq Sa$ .

We know that A is dense in X ,  $\exists t \in A$  within  $\frac{\varepsilon}{3}$  of x. Note that  $s \in B_{\frac{\varepsilon}{3}}(t)$  and that

$$B_{\frac{2\varepsilon}{3}}(t) \subseteq Sa$$

Take  $q \in Q$  such that  $\frac{\varepsilon}{3} < q < \frac{2\varepsilon}{3}$ . Then  $B_q(y) \subseteq B_{\frac{2\varepsilon}{3}}(y) \subseteq Sa$ . Since  $B_q(t)$  has rational radius and center

in A , it is a ball in B and is contained in Sa, it is in the collection C. Thus , every  $s \in X$  belongs to a ball in C. So , C is countable open cover of s. Every ball  $B \in C$  is in at-least one set Sa in  $\{ Sa \}$  . Let  $\alpha$  , such that  $B \subseteq S_{\alpha\beta}$  . Since C is countable and cover X and since  $\{ S_{\alpha\beta} \mid B \in C \}$  covers C,  $\{ S_{\alpha\beta} \mid B \in C \}$  is countable sub-cover of X.

We have shown that it has a countable sub-cover . Now assuming that there is no finite sub-cover . Since  $\{S_n\}$  has no finite sub-cover  $U_{k=1}^n S_k$  does not contain X for every n. X is subset of (X, d) an  $n_i$  such that  $s_1 \in S_{n1}$  .

Let  $s_2 \in X$  , such that  $s_2 \notin U_{k=1}^{n_1} S_k$  . Since  $\{ S_n \}$  covers X , there exist  $n_2$  such that  $s_2 \in S_{n2}$  .

Let  $s_3 \in X$  , such that  $s_3 \notin U_{k=1}^{n_2} S_k$  . Since  $\{ S_n \}$  covers X , there exist  $n_3$  such that  $s_3 \in S_{n3}$

Similarly  $s_k \in S_{nk} \& s_k \notin U_{n=1}^{n_{k-1}} S_n$

So ,  $S_{nk} \neq S_n$  for  $n= 1,2,3,4, \dots , n_{k-1}$  and  $n_k$  is strictly increasing.

Since , (X,d) is sequentially compact so it is the X ,  $\{s_n\}$  must have a sub-sequence that converges to a point  $s_n \in X$  . Since  $\{S_n\}$  covers X ,  $s_1 \in S_n$  for some n. However there exist a  $k_n$  such that  $x_k \notin S_n$  .

$X \in S_n$  , yet the sequence  $(s_n)$  and hence any sub-sequence of  $(s_n)$  can't be in  $S_n$ .

This contradicts that  $(s_n)$  must have a sub-sequence converging to x and the sequential compactness of X .

$\therefore$  the open cover  $\{S_n\}$  must have a finite sub-cover and X must be compact .

EXAMPLE 3.1 : Let (X, D) be an F metric space  $B \subset X$  and  $x \in X$ . Then  $x \in B(\text{compliment}) \Leftrightarrow D(x, B) = 0$ , where  $D(x, B) = \inf D(x, y) \mid y \in B$ .

### CONCLUSION

Fixed-point findings for convex contraction mappings in F-metric spaces were discussed. We also introduced Lindelof metric space to show the compactness of F-metric spaces. As far as we know the work we did herein is fundamental and can be further improved upon when ameliorated in the field of generalized obvious models of F-metric spaces. This work was supported by the Department of Mathematics of Chandigarh University. The authors, therefore are gratefully acknowledge to the DOM of CU for their support.

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